1995

1. One of the roots of the quadratic equation $x^2 + px + 12 = 0$ is one quarter of the other root. Find the possible value of p.

$p = -8$ or 8

2. Given $\frac{1}{2}$ and -5 are the roots of a quadratic equation. Write the quadratic equation in the form of $ax^2 + bx + c = 0$.

$2x^2 + 9x - 5 = 0$

1996

3. (a) Find the values of λ such that the equation $(3-\lambda)x^2 - 2(\lambda+1)x + \lambda + 1 = 0$ has equal roots. Hence, find the roots of the equation based on the values of λ obtained.

(b) Given that the curve $y = 3 + 2x - x^2$ has the equation of tangent in the form $y = mx + 4$.

(i) Calculate the values of m,

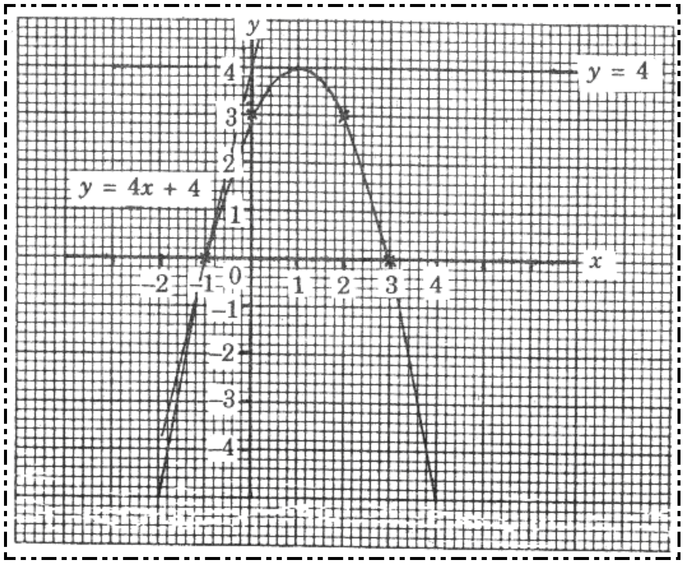
(ii) Draw the graph $y = 3 + 2x - x^2$. On the graph, draw the tangent $y = mx + 4$ based on the values of m obtained in (b)(i).

(a) $\lambda = \pm 1$;
Roots of equation $\lambda = 1, x = 1$
$\lambda = -1, x = 0$

(b) (i) $m = 4, m = 0$

(ii)

x	-2	-1	0	1	2	3	4
y	-5	0	3	4	3	0	-5



1997

4. Given $m + 2$ and $n - 1$ are the roots of the equation $x^2 + 5x = -4$. Find the possible values of m and n.

$x^2 + 5x = -4$
$x^2 + 5x + 4 = 0$
$(x + 4)(x + 1) = 0$
$x = -4$ or $x = -1$

$m + 2 = -4$, $n - 1 = -1$
$m = -6$, $n = 0$
$m + 2 = -1$, $n - 1 = -4$
$m = -3$, $n = -3$

1998

5. The equation $px^2 + px + 3q = 1 + 2x$ has the roots $\frac{1}{p}$ and q.

(a) Find the values of p and q.

(b) Hence, by using the values of p and q in (a), form the quadratic equation which has roots p and $-2q$.

(a) $px^2 + (p - 2)x + 3q - 1 = 0$
Sum of roots
$\frac{1}{p} + q = \frac{-(p-2)}{p}$

$$\frac{1+pq}{p} = \frac{-(p-2)}{p}$$
$$1 + pq = -p + 2$$
$$pq + p = 1 \quad \ldots\ldots (1)$$

Product of roots $\frac{1}{p} \times q = \frac{3q-1}{p}$

$$q = 3q - 1$$
$$2q = 1$$
$$q = \frac{1}{2}$$
$$\therefore p\left(\frac{1}{2}\right) + p = 1$$
$$\frac{3}{2}p = 1$$
$$p = \frac{2}{3}$$

(b) Quadratic equation that have roots p and $-2q$ is

$$x^2 - (p - 2q)x - 2pq = 0$$
$$x^2 - \left[1 - 2\left(\frac{1}{2}\right)\right]x - 2(1)\left(\frac{1}{2}\right) = 0$$
$$x^2 - 0 - 1 = 0$$
$$x^2 - 1 = 0$$

1999

6. One of the roots of the equation $2x^2 + 6x = 2k - 1$ is two times the other root, where k is a constant. Find the roots and the value of k.

$$2x^2 + 6x - 2k + 1 = 0$$

α, 2α are roots

Sum of roots

$$3\alpha = -\frac{6}{2} = -3$$
$$\alpha = -1$$

$\therefore$ roots are -1 and -2

$$2\alpha^2 = \frac{(-2k+1)}{2}$$
$$\frac{(-2k+1)}{2} = 2$$
$$k = -\frac{3}{2}$$

7. (a) Given the equation $x^2 - 6x + 7 = h(2x - 3)$ has equal roots. Find the values of h.

(b) Given α and β are the roots of the equation $x^2 - 2x + k = 0$, while 2α and 2β are the roots of the equation $x^2 + mx + 9 = 0$. Calculate the possible values of k and m.

(a)
$$x^2 - 6x + 7 = h(2x - 3)$$
$$x^2 - x(6 + 2h) + 7 + 3h = 0$$
$$b^2 - 4ac = 0$$
$$(6 + 2h)^2 - 4(7 + 3h) = 0$$
$$36 + 24h + 4h^2 - 28 - 12h = 0$$
$$h^2 + 3h + 2 = 0$$
$$(h + 1)(h + 2) = 0$$
$$h = -1,\ h = -2$$

(b)
$$\alpha + \beta = 2$$
$$\alpha\beta = k$$
$$2\alpha + 2\beta = -m$$
$$2(\alpha + \beta) = -m$$
$$2 \times 2 = -m$$
$$m = -4$$

$$2\alpha \times 2\beta = 9$$
$$4\alpha\beta = 9$$
$$4k = 9$$
$$k = \frac{9}{4}$$

2000

8. The quadratic equation $2x^2 + px + q = 0$ has the roots -4 and 2. Find

(a) the values of p and q.

(b) the range of the values of k so that $2x^2 + px + q = k$ has no real roots.

(a) Sum of roots , $\alpha + \beta = -\frac{b}{a}$

$$-6 + 3 = -\frac{p}{2}$$
$$p = 6$$

Product of roots , $\alpha\beta = \frac{c}{a}$

$$-6(3) = \frac{q}{2}$$
$$q = -36$$

(b) $2x^2 + px + q - k = 0$

$b^2 - 4ac < 0$

$$
\begin{aligned}
p^2 - 4(2)(q - k) &< 0 \\
62 - 8(-36 - k) &< 0 \\
36 + 288 + 8k &< 0 \\
8k &< -324 \\
k &< -40.5
\end{aligned}
$$