1995

1. Given the functions $f : x \to 3x + c$ and the inverse function $f^{-1} : x \to mx + \frac{4}{3}$. Find
   (a) the values of *m* and *c*,
   (b) (i) $f(3)$
   (ii) $f^{-1}f(3)$.

   (a) $m = \frac{1}{3}$, $c = -4$
   (b) (i) 5 (ii) 3

2. Given the functions $f : x \to mx + n$, $g : x \to (x+1)^2$ and $fg : x \to 2(x+1)^2 - 5$. Find
   (a) (i) the value of $g^2(1)$,
   (ii) the values of *m* and *n*,
   (iii) gf^{-1}.

   (a) (i) -3 (ii) $m = 2$, $n = 3$
   (iii) $\frac{x-3}{2}$

1996

3. Given the functions $f : x \to \frac{hx+k}{x-2}$, $x \neq 2$ and the inverse function $f^{-1} : x \to \frac{2x-5}{x-3}$, $x \neq 3$. Find
   (a) the values of *h* and *k*,
   (b) the values of *x* such that $f(x) = 2x$.

   (a) $k = -5$ $h = 3$
   (b) $x = \frac{5}{2}$ or 1

4. Given the function $f : x \to 2x + 5$ and $fg : x \to 13 - 2x$
   (a) (i) Express, in similar form, the function of gf,
   (ii) Find the values of *c* if $gf(c^2 + 1) = 5c - 6$.

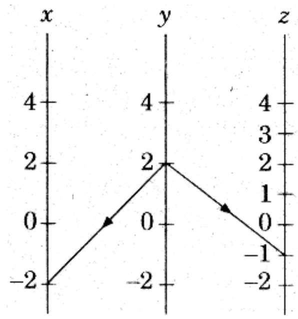
   (a) (i) $gf : x \to -2x - 1$
   (ii) $c = \frac{1}{2}$ or -3

1997

5. Given the function $g : x \to px + q$ and $g^2 : x \to 25x + 48$.
   (a) Find the values of *p* and *q*,
   (b) Assuming p > 0, find the values of *x* so that $2g(x) = g(3x+1)$.

   (a) $g(x) = px + q$
   $g^2(x) = gg(x)$
   $= g(px + q)$
   $= p(px + q) + q$
   $= p^2x + pq + q$
   Given $g^2(x) = 25x + 48$
   hence $p^2 = 25$, $p = \pm 5$
   $pq + q = 48$
   If $p = 5$,
   $5q + q = 48$
   $6q = 48$
   $q = 8$
   If $p = -5$,
   $-5q + q = 48$
   $-4q = 48$
   $q = -12$
   (b) $p > 0$, hence $p = 5$, $q = 8$
   $2g(x) = g(3x + 1)$
   $2(px + q) = p(3x + 1) + q$
   $2px + 2q = 3px + p + q$
   $10x + 16 = 15x + 13$
   $-5x = -3$
   $x = \frac{3}{5}$

6.



The diagram above represents the mapping if *y* onto *x* by the function $g : x \to ay + b$ and the mapping of *y* onto *z* by the function $h : y \to \frac{6}{2y - b}$, $y \neq \frac{b}{2}$. Find

(a) (i) the values of *a* and *b*.

(ii) the function which maps *x* onto *y*.

(iii) the function which maps *x* onto *z*.

(a) (i) $g(y) = ay + b$

$g(2) = 2a + b = -2 \quad \ldots\ldots (1)$

$h(y) = \dfrac{6}{2y - b}$

$h(2) = \dfrac{6}{4 - b} = -1$

$6 = b - 4$

$b = 10 \quad \ldots\ldots (2)$

Substitute $b = 10$ into (1)

$2a + 10 = -2$

$2a = -12$

$a = -6$

(ii) $g^{-1}(x) = y$

$g^{-1}(x) = y$

$g(y) = x$

$-6y + 10 = x$

$y = \dfrac{10 - x}{6}$

$g^{-1} : x \to \dfrac{10 - x}{6}$

(iii) Function that direct *x* to *z* is $hg^{-1}(x)$

$= h(\dfrac{10 - x}{6})$

$= \dfrac{6}{2(\frac{10 - x}{6}) - 10}$

$= \dfrac{36}{(-2x - 40)}$

$= \dfrac{-18}{x + 20}$

1998

7. Given $h(t) = 2t + 5t^2$ and $v(t) = 2 + 9t$. Find

(a) the value of *t* so that $h(t) = 3$,

(b) $h(v)$,

(c) the value of *h(t)* when $v(t) = 110$.

Given $h(t) = 2t + 5t^2$ and

$v(t) = 2 + 9t$

(a) $h(t) = 3$

$2t + 5t^2 = 3$

$5t^2 + 2t - 3 = 0$

$(5t - 3)(t + 1) = 0$

$t = \dfrac{3}{5}$ or $t = -1$

(b) $h[v(t)] = h(2 + 9t)$

$= 2(2 + 9t) + 5(2 + 9t)^2$

$= 4 + 18t + 5(4 + 36t + 81t^2)$

$= 4 + 18t + 20 + 180t + 405t^2$

$= 405t^2 + 198t + 24$

(c) $v(t) = 110$

$2 + 9t = 110$

$9t = 108$

$t = \dfrac{108}{9} = 12$

$h(t) = 2(12) + 5(12)^2$

$= 24 + 5(144)$

$= 24 + 720$

$= 744$

8. Given $f(x) = 6x + 5$ and $g(x) = 2x + 3$. Find

(a) $fg^{-1}(x)$

(b) the value of *x* so that $gf(-x) = 25$.

(a) Given $f(x) = 6x + 5$ and $g(x) = 2x + 3$

(i) Find $g^{-1}(x)$ first .

If $g^{-1}(x) = y$

Hence $g(y) = x$

$2y + 3 = x$

$y = \dfrac{x - 3}{2}$

Hence $g^{-1}(x) = \frac{x-3}{2}$

So, $fg^{-1}(x) = f\left(\frac{x-3}{2}\right)$

$= 6\left(\frac{x-3}{2}\right) + 5$

$= 3(x-3) + 5$

$= 3x - 9 + 5$

$= 3x - 4$

(ii) Given $gf(-x) = 25$

hence $g[6(-x) + 5] = 25$

$g(5 - 6x) = 25$

$2(5 - 6x) + 3 = 25$

$10 - 12x + 3 = 25$

$13 - 12x = 25$

$12x = 13 - 25 = -12$

$x = -1$

1999

9. Given $f : x \to k - mx$. Find
   (a) $f^{-1}(x)$ in terms of k and m,
   (b) the values of k and m, if $f^{-1}(14) = -4$ and $f(5) = -13$.

(a) $f(x) = k - mx$

$k - mx = y$

$x = \frac{k-y}{m}$

$f^{-1}(x) = \frac{k-x}{m}$

(b) $f^{-1}(14) = -4$

$\frac{k-14}{m} = -4$

$k + 4m = 14$ .... ①

$f(5) = -13 = k - 5m$

$k - 5m = -13$ .... ②

Solve the simultaneous equation ① and ②

$m = 3,\ k = 2$

10. The function g is defined by $g : x \to x + 3$.
    While the function f is such that $fg : x \to x^2 + 6x + 7$. Find
    (i) the function of $f(x)$,
    (ii) the value of k if $f(2k) = 5k$.

$g(x) = x + 3$

$fg(x) = x^2 + 6x + 7$

(i) $f(x + 3) = x^2 + 6x + 7$

$= (x + 3)^2 - 2$

$\therefore f(x) = x^2 - 2$

(ii) $f(2k) = 4k^2 - 2 = 5k$

$4k^2 - 5k - 2 = 0$

$k = \frac{5 + \sqrt{57}}{8}, \frac{5 - \sqrt{57}}{8}$

$k = 1.57, -0.32$

2000

11. Given $g^{-1}(x) = \frac{5 - kx}{3}$ and $f(x) = 3x^2 - 5$.
    Find
    (a) $g(x)$,
    (b) the values of k so that $g(x^2) = 2f(-x)$.

(a) If $(y) = x$

hence $g^{-1}(x) = y$

$\frac{5 - kx}{3} = y$

that is $-kx = 3y - 5$

$x = \frac{5 - 3y}{k}$

$= \frac{5 - 3y}{k}$

Hence $g(x) = \frac{5 - 3x}{k}$

(b) $g(x^2) = 2f(-x)$

$\frac{5 - 3x^2}{k} = 2[3(-x)^2 - 5]$

$k = \frac{5 - 3x^2}{2(3x^2 - 5)}$

$= -\frac{1}{2}$

12. Given the function $f : x \to 4 - 3x$.
    (a) Find
    (i) $f^2(x)$,
    (ii) $(f^2)^{-1}(x)$.

(b) Hence, or otherwise, find $(f^{-1})^2(x)$ and show that $(f^2)^{-1}(x) = (f^{-1})^2(x)$.

(a) $f(x) = 4 - 3x$

(i) $f^2(x) = f(4 - 3x)$

$= 4 - 3(4 - 3x)$

$= 9x - 8$

(ii) If $f^2(x) = y$

hence $(f^2)^{-1}(y) = x$

$9x - 8 = y$

$x = \dfrac{y + 8}{9}$

$(f^2)^{-1}(y) = \dfrac{y + 8}{9}$

that is $(f^2)^{-1}(x) = \dfrac{x + 8}{9}$

(b) $f(x) = 4 - 3x$

$f^{-1}(x) = \dfrac{4 - x}{3}$

$(f^{-1})^2(x) = \dfrac{4 - \dfrac{(4 - x)}{3}}{3}$

$= \dfrac{12 - 4 + x}{9}$

$= \dfrac{8 + x}{9}$

Hence $(f^2)^{-1}(x) = (f^{-1})^2(x)$